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Radiative mass generation and suppression of supersymmetric contributions to flavor changing processes

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We explore the possibility that the masses for the first two generations of fermions and the quark flavor violation are generated radiatively in the Minimal Supersymmetric Standard Model. We assume that the source of all flavor violation resides in the the supersymmetry breaking sector and is transmitted radiatively to the Standard Model fermion sector through finite corrections at low energy. The approximate radiative alignment between the Yukawa and soft supersymmetry breaking matrices, helps to suppress some of the supersymmetric contributions to flavor changing processes, overcoming current experimental constraints. This mechanism may also explain the non-observation of proton decay, since flavor conservation in the superpotential would imply the suppression of dimension five operators in supersymmetric grand unified theories.

I. INTRODUCTION

An outstanding unsolved problem in the Standard Model is the origin of the fermion mass hierarchies. A related puzzle is the origin of the flavor violation observed in the quark sector. These two problems appear to be connected, since the fermion mass hierarchies cannot be explained with precision without a theory of flavor.

Many models have been proposed to explain the fermion mass hierarchies and the quark mixing angles. Some of the most popular are mass-matrix texture models, higher-order non-renormalizable operators, horizontal symmetries, and fixed point mechanisms. While some of the proposed theories can fit some of the experimental data they lack a convincing and predictive basic principle to explain the origin of the fermion hierarchies. Fixed point models, in particular, have provided an explanation for the top quark mass [1] but lack predictivity in a realistic three-generation model.

There is another possibility, which has not received much attention lately. The fermion mass hierarchies suggest that the masses of the lighter fermions may arise only as higher-order radiative effect. Following the original suggestion by S. Weinberg [2, 3] of a mechanism to generate the electron mass radiatively from a tree-level muon mass, several proposals in the framework of non-supersymmetric models were published. The program was considered more difficult to implement in the context of supersymmetric (*hereafter* SUSY) models, since, as pointed out by L. Ibañez, if supersymmetry is spontaneously broken only tiny fermion masses can be generated radiatively [4]. A few ideas to alleviate this problem have been proposed. Especially interesting is the possibility, originally suggested by W. Buchmuller and D. Wyler [5] and later rediscovered in Refs. [6, 7, 8, 9], that the presence of soft supersymmetry breaking terms allows for the radiative generation of quark and charged lepton masses through sfermion–gaugino loops. The gaugino mass contributes the violation of fermionic chirality required by a fermion mass, while the soft breaking terms provide the necessary violation of chiral flavor symmetry. (Additional implications of this possibility have been studied in Refs. [10, 11, 12, 13, 14]).

In this paper I will analyze, in the context of the minimal supersymmetric Standard Model (*hereafter* MSSM), the possibility that the masses for the first two generations of fermions, as well as the observed mixing in the quark sector, can actually be generated radiatively. I will use to this end low-energy one-loop finite SUSY threshold corrections coming from flavor violating mixings in the soft supersymmetry breaking sector. I will study the constraints on the flavor violating soft breaking sector and on the supersymmetric spectra imposed by the experimental data on masses, mixings and flavor changing processes. The basic conditions that one can expect from a unified supersymmetric theory, which provides the MSSM boundary conditions at a higher energy scale, generating fermion masses radiatively are,

1. A symmetry or symmetries of the superpotential guarantee flavor conservation and precludes tree-level masses for the first and second generations of fermions in the supersymmetric limit.

2. The supersymmetry breaking terms receive small corrections, which violate the symmetry of the superpotential and are responsible for the observed flavor physics.

Under the first assumption, one expects the Yukawa matrices provided as a boundary condition for the MSSM at some high energy scale to be of the form,

$$\mathbf{Y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y \end{bmatrix}, \tag{1}$$

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where $\mathbf{Y}$ stands generically for the 3×3 quark and lepton Yukawa matrices. We observe that the structure of the Yukawa matrices given by Eq. 1 is independent of the renormalization scale. Thus, the renormalization group running from the unification scale down to the electroweak scale cannot generate non-zero entries. The opposite is not true: flavor violation, if present in the Yukawa matrices, would transmit to the soft sector through renormalization group running. Under the second assumption, one expects the soft trilinear matrices generically to look like,

$$\mathbf{A} = A\mathcal{O}(\lambda), \tag{2}$$

where A is a scalar number and $\mathcal{O}(\lambda)$ represents generically a dimensionless flavor violating polynomial matrix expanded in powers of λ. The flavor violating perturbation parameter, λ, is expected to be determined *a posteriori* by the ratios between the fermion masses. It is known that in an effective field theory format, holomorphic trilinear soft supersymmetry-breaking terms originate via non-renormalizable operators that couple to the supersymmetry-breaking chiral superfields, $\mathcal{Z}$, and are suppressed by powers of the messenger scale M. These operators would be generically of the form,

$$\frac{1}{M}\int d^2\theta \mathcal{Z} H_\alpha \phi^L \phi^R + c.c., \tag{3}$$

where $\mathcal{Z} = \mathcal{Z}_s + \mathcal{Z}_a\theta^2$. The vacuum expectation value (*hereafter* vev) of the auxiliary component, $\langle \mathcal{Z}_a \rangle$, parametrizes the supersymmetry breaking scale, M_S^2. If we assume that the vev of the scalar component of $\mathcal{Z}$ vanishes, $\langle \mathcal{Z}_s \rangle = 0$, or is much smaller than the messenger scale, $\langle \mathcal{Z}_s \rangle \ll M$, then no Yukawa couplings would arise but only trilinear soft breaking terms. These conditions could be enforced by an O'Raifeartaigh type model superpotential or by continuous or discrete horizontal flavor R–symmetries. Flavor violation in the soft terms may arise, for instance, if the supersymmetry-breaking sector fields, $\mathcal{Z}$, transform non-trivially under flavor symmetries. In principle, one also expects flavor violating contributions to the soft mass matrices arising from operators generically of the form,

$$\frac{1}{M^2}\int d^4\theta \mathcal{Z}\mathcal{Z}^\dagger k^2 \phi^{(L,R)}\phi^{(L,R)\dagger}; \tag{4}$$

here flavor indices have been omitted and k^2 is a dimensionless parameter determined by the underlying supersymmetric theory. The magnitude of flavor violation in the soft mass matrices would depend on the particular model of flavor. Henceforth I will assume that the flavor violation is concentrated in the soft trilinear matrices while the left and right handed soft mass matrices are flavor conserving,

$$\mathcal{M}^2_{D_{(L,R)}} = m^2_{\widetilde{b}} \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

where $m_{\widetilde{b}} \simeq k \langle \mathcal{Z}_a \rangle / M$. To add more generality to the analysis I will also assume that there is squark non-degeneracy in the left and right handed sectors given by the parameter σ. This coefficient measures the non-degeneracy between the sfermion masses of different generations, $\sigma = \widetilde{m}_{1,2}/\widetilde{m}_3$. A realistic flavor model based on a $\mathrm{SU}(5)_V \times \mathrm{U}(2)_H$ symmetry which implements the MSSM boundary conditions as described above [15]. We must note that the assumption of flavor conservation in the soft mass matrices and of approximate degeneracy between first and second generation sfermion masses is important to overcome Δm_K and ϵ_K constraints. We will comment with more detail on this remark later. In the rest of this paper a phenomenological analysis of this basic scenario will be implemented, independently of the particular model that provides these MSSM boundary conditions. I will show that in this scenatio is possible to fit the fermion mass ratios, the CKM matrix elements and overcome constraints on flavor changing processes.

II. RADIATIVE GENERATION OF YUKAWA COUPLINGS

In the presence of flavor violation in the soft sector, the left and right handed components of the sfermions mix. For instance, in the gauge basis the 6×6 down–type squarks mass matrix is given by,

$$\mathcal{M}^2_D = \begin{bmatrix} \mathcal{M}^2_{D_L} + v^2 c^2_\beta \mathbf{Y}^\dagger_D \mathbf{Y}_D & (\mathbf{A}^\dagger_D c_\beta - \mu \mathbf{Y}_D s_\beta)v \\ (\mathbf{A}_D c_\beta - \mu \mathbf{Y}^\dagger_{\mathbf{D}} s_\beta)v & \mathcal{M}^2_{D_R} + v^2 c^2_\beta \mathbf{Y}_D \mathbf{Y}^\dagger_D \end{bmatrix}, \tag{6}$$

where $\mathcal{M}^2_{D_R}$ and $\mathcal{M}^2_{D_L}$ are the 3×3 right handed and left handed soft mass matrices (including D-terms), $\mathbf{A}_D$ is the 3×3 soft trilinear matrix, $\mathbf{Y}_D$ is the 3×3 tree-level Yukawa matrix, $\tan\beta$ is ratio of Higgs expectation values in the MSSM, μ is the so-called mu-term (which is allowed in the superpotential) and $v = s_W m_W/\sqrt{2\pi\alpha_e} =$ 174.5 GeV. $\mathcal{M}^2_D$ is diagonalized by a 6×6 unitary matrix, $\mathcal{Z}^D$. In general, the dominant finite one-loop contribution to the 3×3 down–type quark Yukawa matrix is given by the gluino-squark loop,

$$(\mathbf{Y}_D)^{\mathrm{rad}}_{ab} = \frac{\alpha_s}{3\pi} m^*_{\widetilde{g}} \sum_c \mathcal{Z}^D_{ac}\mathcal{Z}^{D*}_{(b+3)c} B_0(m_{\widetilde{g}}, m_{\widetilde{d}_c}), \tag{7}$$

where $\widetilde{d}_c$ $(c = 1, \cdots, 6)$ are mass eigenstates and $m_{\widetilde{g}}$ is the gluino mass. B_0 is a known function defined in the appendix. The radiatively corrected 3×3 down–type quark mass matrix is given by,

$$\mathbf{m}_D = v c_\beta (\mathbf{Y}_D + \mathbf{Y}^{\mathrm{rad}}_D). \tag{8}$$

If the supersymmetric spectra were degenerate, i.e. $\sigma = 1$, one would use the mass insertion method to obtain a simple expression for $\mathbf{Y}^{\mathrm{rad}}_D$,

$$\mathbf{Y}^{\mathrm{rad}}_D = \frac{2\alpha_s}{3\pi} m^*_{\widetilde{g}} (\mathbf{A}_D + \mu \mathbf{Y}_D \tan\beta) F(m_{\widetilde{b}}, m_{\widetilde{b}}, m_{\widetilde{g}}), \tag{9}$$

where the function $F(x,y,z)$ is given by,

$$F(x,y,z)=\frac{\left[(x^2y^2\ln\frac{y^2}{x^2}+y^2z^2\ln\frac{z^2}{y^2}+z^2x^2\ln\frac{x^2}{z^2}\right]}{(x^2-y^2)(y^2-z^2)(z^2-x^2)}>0. \tag{10}$$

We will ignore CP-violating phases from now on. We note that in the degenerate squark limit, if it were not for tree level contribution to the third generation fermion masses, the radiatively generated Yukawa matrices would be aligned with the soft trilinear matrices. In the general case, the way the non–degeneracy in the squark spectra affects the predictions is going to depend on the flavor structure in the soft trilinear matrices.

To make my case I will pick a simple texture for the soft trilinear matrix $\mathbf{A}_D$, which is motivated on a model with a horizontal $\mathrm{U}(2)_H$ symmetry [15, 16]. By simple I mean having only one flavor violating parameter [17],

$$\mathbf{A}_D=A_b\begin{bmatrix}0 & \lambda_d^2 & \lambda_d^2\\ \lambda_d^2 & \lambda_d & \lambda_d\\ \lambda_d^2 & \lambda_d & 1\end{bmatrix}, \tag{11}$$

The analysis that will follow can be easily extended to other possible textures. Let us assume that squark masses of the first and second generations are degenerate. In this case one obtains a simple expression for the radiatively corrected down–type quark mass matrix,

$$\mathbf{m}_D=\widehat{m}_b\begin{bmatrix}0 & \gamma_b\lambda_d^2 & \kappa\gamma_b\lambda_d^2\\ \gamma_b\lambda_d^2 & \gamma_b\lambda_d & \kappa\gamma_b\lambda_d\\ \kappa\gamma_b\lambda_d^2 & \kappa\gamma_b\lambda_d & 1\end{bmatrix}, \tag{12}$$

where κ is a squark non-degeneracy coefficient, which in the $m_{\widetilde{q}}\geq m_{\widetilde{g}}$ limit is given by,

$$\kappa=\kappa_{13}^d=\kappa_{31}^d=\kappa_{23}^d=\kappa_{32}^d=\sigma^2\ln\sigma^2/(\sigma^2-1). \tag{13}$$

In general κ_{ij}^d is defined by,

$$\kappa_{ij}^d=\frac{F(m^2_{(\widetilde{d}_L)_i},m^2_{(\widetilde{d}_R)_j},m^2_{\widetilde{g}})}{F(m^2_{(\widetilde{d}_L)_2},m^2_{(\widetilde{d}_R)_2},m^2_{\widetilde{g}})}, \tag{14}$$

Here σ is the ratio between first/second and third generation down–type squarks introduced in Eq. 5,

$$\widehat{m}_b=vc_\beta\left(y_b+\rho_D(1-\frac{\mu}{A_b}y_b\tan\beta)\right), \tag{15}$$

and

$$\gamma_b=\frac{vc_\beta\rho_D}{\widehat{m}_b}. \tag{16}$$

γ_b parametrizes the breaking of the alignment between soft trilinear and the Yukawa sector caused by the presence of a tree-level mass for the bottom quark, and ρ_D encodes the dependence on the supersymmetric spectra. For the case $m_{\widetilde{q}}\geq m_{\widetilde{g}}$ and $\sigma\lesssim 2$ one obtains,

$$\rho_D=\frac{2\alpha_s}{3\pi}\left(\frac{m_{\widetilde{g}}}{m_{\widetilde{b}}}\right)\left(\frac{A_b}{m_{\widetilde{b}}}\right)\left(\frac{1}{\sigma^2}\right), \tag{17}$$

(for $\sigma>2$ one should substitute $\sigma\to 2\ln\sigma$ in the Eq. 17). Although not diagonal in the gauge basis, the matrix $\mathbf{m}_D$ can be brought to diagonal form in the mass basis by a biunitary diagonalization, $(\mathcal{V}_L^d)^\dagger\mathbf{m}_D\mathcal{V}_R^d=(m_d,m_s,m_b)$. The down–type quark mass matrix given by Eq. 12 makes the following predictions for the quark mass ratios,

$$\frac{m_d}{m_s} = \lambda_d^2(1+2\kappa^2\gamma_b\lambda_d-2\lambda_d^2)+\mathcal{O}(\lambda_d^4), \tag{18}$$

$$\frac{m_s}{m_b} = \gamma_b\lambda_d(1-\kappa^2\gamma_b\lambda_d+\lambda_d^2)+\mathcal{O}(\lambda_d^3), \tag{19}$$

These formulas approximately reduce to,

$$\frac{m_d}{m_s}=\lambda_d^2,\quad \frac{m_s}{m_b}=\gamma_b\lambda_d. \tag{20}$$

We can relate λ_d and γ_b with dimensionless and approximately renormalization scale independent fermion mass ratios. To first order,

$$\lambda_d=\left(\frac{m_d}{m_s}\right)^{1/2},\quad \gamma_b=\left(\frac{m_s^3}{m_b^2m_d}\right)^{1/2}, \tag{21}$$

Using these relations and the running quark masses determined from experiment (see appendix), we can determine λ_d and γ_b. In the degenerate squark mass limit, i.e. $\kappa=1$, we obtain,

$$\lambda_d = 0.209\pm0.019, \tag{22}$$

$$\gamma_b = 0.109\pm0.030, \tag{23}$$

We observe that the size of the flavor violating corrections to the soft breaking terms is determined by quark mass ratios, while constraints on the supersymmetric spectra can be derived from the parameter γ_b. One further observe that for this texture the squark non-degeneracy affects the determination of λ_d and γ_b at the next order in $\gamma_b\lambda_d$. From Eqs. 16 & 17 we obtain the following upper bound on the down–type squark non-degeneracy,

$$\sigma=\frac{1}{\gamma_b}\left(\frac{vc_\beta}{m_b}\right)\left(\frac{2\alpha_s}{3\pi}\right)\left(\frac{m_{\widetilde{g}}}{m_{\widetilde{d}}}\right)\left(\frac{A_b}{m_{\widetilde{b}}}\right)\lesssim\frac{30}{t_\beta}, \tag{24}$$

where I used $\gamma_b=0.1$, $v=174.5$ GeV, $\alpha_s=0.117$, $m_{\widetilde{q}}>m_{\widetilde{g}}$ and $A_b<2m_{\widetilde{b}}$. This last condition, $A_b<2m_{\widetilde{b}}$, approximately guarantees the stability of the scalar potential. We conclude from Eq. 24 that it is possible to fit the quark mass ratios with an arbitrary squark spectra, except for large $\tan\beta$ where the condition $m_{\widetilde{q}_{1,2}}\lesssim m_{\widetilde{b}}$ (compatible with a degenerate squark spectra) is required. In the limit $m_{\widetilde{g}}>2m_{\widetilde{q}}$ one obtains,

$$\gamma_b=\frac{2\alpha_s}{3\pi}\left(\frac{vc_\beta}{m_b}\right)\left(\frac{A_b}{m_{\widetilde{g}}}\right)\ln\left(\frac{m_{\widetilde{b}}}{m_{\widetilde{g}}}\right)\lesssim\frac{1.5}{t_\beta}. \tag{25}$$

This possibility is perfectly viable for low $\tan\beta$ and for large $\tan\beta$. For instance, if $\tan\beta>25$ one obtains $\gamma_b<0.05$, which is still compatible with γ_b being a loop factor.

In the up–type quark sector, one can perform a similar analysis. Let us assume the following particular texture as a case study,

$$\mathbf{A}_U = A_t \begin{bmatrix} \lambda_u^6 & 0 & 0 \\ 0 & \lambda_u^2 & -\lambda_u \\ 0 & -\lambda_u & 1 \end{bmatrix}, \tag{26}$$

which is inspired on a SU(5) unified model with a horizontal $U(2)_H$ horizontal symmetry [15, 16]. Our choice of sign in the entry (23) will be clear later when we calculate the CKM mixing matrix. One can obtain a simple expression for the radiatively corrected up–type quark mass matrix including squark non–degeneracy,

$$\mathbf{m}_U = \widehat{m}_t \begin{bmatrix} \gamma_t\lambda_u^6 & 0 & 0 \\ 0 & \gamma_t\lambda_u^2 & -\kappa\gamma_t\lambda_u \\ 0 & -\kappa\gamma_t\lambda_u & 1 \end{bmatrix}, \tag{27}$$

where $\kappa = \kappa_{23}^u = \kappa_{32}^u$ is the up–type squark non–degeneracy coefficient, that we assume to simplify is the same that as in the down-type squark sector,

$$\widehat{m}_t = v s_\beta \left(y_t + \rho_U (1 - \frac{\mu}{A_t} y_t \cot\beta) \right), \tag{28}$$

and

$$\gamma_t = \frac{v s_\beta \rho_U}{\widehat{m}_t}. \tag{29}$$

ρ_U, in the case $m_{\tilde{q}} \geq m_{\tilde{g}}$ and $\sigma \lesssim 2$, is given by,

$$\rho_U = \frac{2\alpha_s}{3\pi} \left(\frac{m_{\tilde{g}}}{m_{\tilde{t}}}\right) \left(\frac{A_t}{m_{\tilde{t}}}\right) \left(\frac{1}{\sigma^2}\right). \tag{30}$$

After diagonalization, $(\mathcal{V}_L^u)^\dagger \mathbf{m}_U \mathcal{V}_R^u = (m_u, m_c, m_t)$, one obtains the following predictions for the up–type quark mass ratios,

$$\frac{m_u}{m_c} = \lambda_u^4 (1 + \kappa^2 \gamma_t (1 + \gamma_t)) + \mathcal{O}(\gamma_t^2 \lambda_u^6), \tag{31}$$

$$\frac{m_c}{m_t} = \gamma_t \lambda_u^2 (1 - \kappa^2 \gamma_t)(1 - 2\kappa^2 \gamma_t^2 \lambda_u^2) + \mathcal{O}(\lambda_u^6). \tag{32}$$

These approximately reduce to,

$$\frac{m_u}{m_c} = \lambda_u^4, \quad \frac{m_c}{m_t} = \gamma_t \lambda_u^2. \tag{33}$$

We can relate λ_u and γ_t with dimensionless fermion mass ratios, to first order,

$$\lambda_u = \left(\frac{m_u}{m_c}\right)^{1/4}, \quad \gamma_t = \left(\frac{m_c^3}{m_t^2 m_u}\right)^{1/2}, \tag{34}$$

using the invariant running quark mass ratios determined from experiment (see appendix). In the degenerate limit,

$$\lambda_u = 0.225 \pm 0.015, \tag{35}$$

$$\gamma_t = 0.071 \pm 0.018, \tag{36}$$

From Eqs. 29 & 30 we obtain the following upper bound on the up–squarks non-degeneracy,

$$\sigma = \frac{1}{\gamma_t} \left(\frac{v s_\beta}{\widehat{m}_t}\right) \left(\frac{2\alpha_s}{3\pi}\right) \left(\frac{m_{\tilde{g}}}{m_{\tilde{u}}}\right) \left(\frac{A_t}{m_{\tilde{t}}}\right) \lesssim 0.6. \tag{37}$$

This implies that a certain amount of non–degeneracy in the up–type squark sector seems to be required, $m_{\tilde{t}} > 1.5 m_{\tilde{u}}$, to account for the observed quark mass ratios. One may also notice a similarity in the values for λ_d and λ_u in Eqs. 22 & 35 and γ_b and γ_t in Eqs. 23 & 36, which reflects the curious empirical fact,

$$\left(\frac{m_d}{m_s}\right) \approx \left(\frac{m_u}{m_c}\right)^{1/2} \tag{38}$$

$$\left(\frac{m_s^3}{m_b^2 m_d}\right) \approx \left(\frac{m_c^3}{m_t^2 m_u}\right). \tag{39}$$

This may be considered as an experimental evidence pointing toward a common underlying mechanism generating the up and down–type quark mass matrices. An alternative and simpler choice of soft trilinear textures that makes, to leading order in λ, the same predictions for quark mass ratios is the following,

$$\mathbf{A}_D = A_b \begin{bmatrix} 0 & \lambda_d^2 & \lambda_d^2 \\ \lambda_d^2 & \lambda_d & 2\lambda_d \\ \lambda_d^2 & 2\lambda_d & 1 \end{bmatrix}, \tag{40}$$

together with,

$$\mathbf{A}_U = A_t \begin{bmatrix} \lambda_u^6 & 0 & 0 \\ 0 & \lambda_u^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{41}$$

We will see next that this alternative solution makes also the same prediciont for the CKM matrix elements to leading order in λ.

A. Radiatively generated CKM matrix

Finally, one can calculate the CKM mixing matrix, $\mathcal{V}_{CKM} = \mathcal{V}_L^{u\dagger} \mathcal{V}_L^d$. We can express the quark Yukawa diagonalization matrices as a function of the $\gamma_{u,d}$ and $\lambda_{u,d}$ parameters. Assuming the textures given by Eqs. 12 and 27 I obtain to leading order in $\lambda_{u,d}$,

$$|\mathcal{V}_{CKM}^{\rm theo}| = \begin{bmatrix} 1 - \frac{1}{2}\lambda_d^2 & -\lambda_d & \gamma_b \lambda_d^2 \\ \lambda_d & 1 - \frac{1}{2}\left(\lambda_d^2 + \gamma_{ud}^2\right) & -\gamma_{ud} \\ \gamma_t \lambda_u \lambda_d & \gamma_{ud} & 1 - \frac{1}{2}\gamma_{ud}^2 \end{bmatrix}, \tag{42}$$

where,

$$\gamma_{ud} = (\gamma_t \lambda_u + \gamma_b \lambda_d). \tag{43}$$

We observe that flavor violation in the entries (12), (21) and (13) of up–type quark sector is poorly determined from experiment, since λ_u only affects to second order

the corresponding entries of the CKM matrix. Moreover for the texture under consideration $|V_{td}|$ is of the same order than $|V_{ub}|$ and it will not appear if not for the flavor mixing in the up-type quark sector. We can write a simple expression for the predicted CKM matrix at second order if we simplify and assume that $\lambda \approx \lambda_d \approx \lambda_u$ and $\gamma \approx \gamma_t \approx \gamma_b$. We obtain,

$$\left[\begin{array}{ccc} 1-\frac{1}{2}\lambda^2 & \gamma\left(1-\frac{3}{2}\gamma\lambda\right) & \gamma\lambda^2\left(1+\gamma\lambda\right)) \\ \lambda\left(1-\frac{3}{2}\gamma\lambda\right) & 1-\frac{1}{2}\lambda^2\left(1+4\gamma^2\right) & 2\gamma\lambda\left(1+\gamma\lambda\right) \\ \gamma\lambda^2\left(1-\frac{5}{2}\gamma\lambda\right) & 2\gamma\lambda\left(1+\gamma\lambda\right) & 1-2\gamma^2\lambda^2 \end{array}\right], \tag{44}$$

Using the experimentally determined values for γ_b, γ_t, λ_d and λ_u in Eqs. 22, 23, 35 & 36 I obtain the following central theoretical prediction for the CKM matrix, $\left|\mathcal{V}_{CKM}^{\rm theo}\right|$, in the squark degenerate limit, $\kappa = 1$,

$$\left[\begin{array}{ccc} 0.977\pm0.007 & 0.20\pm0.03 & 0.0039\pm0.0006 \\ 0.20\pm0.03 & 0.976\pm0.008 & 0.047\pm0.024 \\ 0.005\pm0.003 & 0.047\pm0.024 & 0.9988\pm0.0012 \end{array}\right], \tag{45}$$

which should be compared with the 90 % C.L. experimental compilation, $\left|\mathcal{V}_{CKM}^{\rm exp}\right|$,

$$\left[\begin{array}{ccc} 0.97485\pm0.00075 & 0.2225\pm0.0035 & 0.00365\pm0.00115 \\ 0.2225\pm0.0035 & 0.9740\pm0.0008 & 0.041\pm0.003 \\ 0.0009\pm0.005 & 0.0405\pm0.0035 & 0.99915\pm0.00015 \end{array}\right]. \tag{46}$$

It appears that there is a very good agreement with the experimental data on CKM matrix elements from the PDG experimental compilation. It is remarkable that such a simple texture can be so close to the experimental results. The scenario can account for the observed quark mass ratios and mixing angles in a natural way, i.e. without adjusting any supersymmetric parameter or without resorting to a highly non–degenerate squark spectra. We note that assuming instead the textures given by the Eqs. 47 and 41 we obtain the same prediction for the CKM matrix to leading order in λ. We noticed also that the kind of textures here considered, *i.e.* with common γ coefficients of order 0.09, consistent with γ being a loop factor, in all the entries of the quark mass matrices except the (33), have not been considered before. The search of Yukawa textures has focused in the past in polynomial matrices in powers of λ with coefficients of order 1.

I would like to point out that the constraints on the supersymmetric spectra are very texture-dependent, which is positive for the testability of this scenario. From now on I will use $\lambda = \lambda_u = \lambda_d$. For instance, assuming a different texture for the soft trilinear matrix $\mathbf{A}_D$,

$$\mathbf{A}_D = A_b \left[\begin{array}{ccc} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{array}\right], \tag{47}$$

the predictions for the down–type quark mass would be,

$$\frac{m_d}{m_s} = \lambda^2 + \mathcal{O}(\lambda^4), \tag{48}$$

$$\frac{m_s}{m_b} = \gamma_b\lambda^2 + \mathcal{O}(\lambda^4), \tag{49}$$

The determination of λ from measured fermion masses would not change at leading order in λ but for γ_b one would obtain $\gamma_b \simeq 0.44$. This is a value much larger than the one we obtained for the texture previously considered and not compatible with γ being a loop factor. This case would imply a considerable amount of non–degeneracy in the down–type squark sector, which would make difficult to fit the CKM elements except for very low $\tan\beta$.

B. Higher order Yukawa couplings

Yukawa couplings which are not generated at one loop could be generated at higher orders. For instance, the Yukawa coupling $(\mathbf{Y}_U)_{13}$ could be generated at two loops through a diagram with gluino and Higgs exchange and three soft trilinear vertices: $(\mathbf{A}_D)_{12}$, $(\mathbf{A}_D)_{22}$ and $(\mathbf{A}_U)_{23}$ [18]. We are interested in an overestimation of this 2-loop Yukawa coupling. Assuming that all the sparticles in the loop have masses of the same order, to maximize the loop factor, we obtain

$$(\mathbf{Y}_U)_{13}^{2-\rm loop} \simeq \left(\frac{2\alpha_s}{3\pi}\right)\left(\frac{1}{4\pi}\right)^2\left(\frac{v}{m_{\tilde{q}}}\right)^2 c_\beta^2\lambda^4, \tag{50}$$

here $v = 175$ GeV. The ratio $v/m_{\tilde{q}}$, the c_β factors ($c_\beta = \cos\beta$) and the λ factors come from the three soft trilinear vertices. To facilitate the comparison with the one-loop generated Yukawa couplings we will express this in powers of λ. Using that $\lambda \simeq 0.2$ and $\gamma \simeq 0.1$, we obtain,

$$(\mathbf{Y}_U)_{13}^{2-\rm loop} \simeq \frac{\gamma\lambda^{10}}{\tan^2\beta}\left(\frac{1\ \rm TeV}{m_{\tilde{q}}}\right)^2. \tag{51}$$

We note that this 2-loop generated Yukawa coupling is very suppressed when compared with the one-loop generated couplings and for all practical purposes it can be considered zero.

III. SUPPRESSION OF FCNCS PROCESSES BY RADIATIVE ALIGNMENT

Overcoming the present experimental constraints on supersymmetric contributions to flavor changing neutral current processes (FCNCs) is a necessary requirement for the consistency of any supersymmetric model [19]. Correlations between radiative mass generation and dipole operator phenomenology were first pointed out in Ref. [20]. For calculational purposes it is convenient to rotate the squarks to the so-called superKM basis, the basis where gaugino vertices are flavor diagonal [21]. The soft trilinear matrix $\mathbf{A_D}$ in the superKM basis is given by,

$$\mathbf{A}_D^{\rm SKM} = (\mathcal{V}_L^d)^\dagger \mathbf{A}_D \mathcal{V}_R^d. \tag{52}$$

Assuming the soft trilinear texture from the previous section, Eq. 11, one obtains, at leading order in λ and γ_b,

$$\left|\mathbf{A}_D^{\mathrm{SKM}}\right| = A_b \left[\begin{array}{ccc} \lambda^3 & \gamma_b\lambda^5 & \lambda^4 \\ \gamma_b\lambda^5 & \lambda & (\gamma_b-1)\lambda \\ \lambda^4 & (\gamma_b-1)\lambda & 1+2\gamma_b\lambda^2 \end{array}\right]. \quad (53)$$

Using for λ_d and γ_b the values given by Eqs. 22 & 23 the amount of soft flavor violation required to fit quark masses and mixing angles is determined,

$$\left|\mathbf{A}_D^{\mathrm{SKM}}\right| = A_b \left[\begin{array}{ccc} 9\times10^{-3} & 4\times10^{-5} & 2\times10^{-3} \\ -- & 0.208 & 0.194 \\ -- & -- & 1.009 \end{array}\right]. \quad (54)$$

The entry most constrained experimentally in the soft trilinear mass matrix is the entry (12). Its contribution to the K_L–K_S mass difference is, following Ref. [21], given by,

$$\Delta m_K = \frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\frac{2}{3} m_K f_K^2 (\delta_{12}^d)_{LR}^2 \left[\left(\frac{m_K}{m_s+m_d}\right)^2 \times \right.$$
$$\left. (268\, xf(x) + 144\, g(x)) + 84\, g(x)\right], \quad (55)$$

where $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, $m_{\tilde{q}} = \sqrt{m_{\tilde{d}_L} m_{\tilde{d}_R}}$ is an average squark mass, $m_K = 497.6$ MeV, $f_K = 160$ MeV, $\alpha_s = 0.117$ and the functions $f(x)$ and $g(x)$ are defined in the appendix. $(\delta_{12}^d)_{LR}$ is given by,

$$(\delta_{12}^d)_{LR} = \frac{v c_\beta (\mathbf{A}_D^{\mathrm{SKM}})_{12}}{m_{\tilde{q}}^2}. \quad (56)$$

Using the predicted flavor violation, Eq. 54, for the soft trilinear texture under consideration,

$$(\delta_{12}^d)_{LR} = 7\times10^{-6}\left(\frac{A_b}{m_{\tilde{q}}}\right)\left(\frac{1\ \mathrm{TeV}}{m_{\tilde{q}}}\right)\frac{1}{t_\beta}. \quad (57)$$

Assuming $\tan\beta > 5$, $A_b < 4 m_{\tilde{q}}$, $m_{\tilde{q}} > 400$ GeV and any gluino-squark mass ratio one obtains a contribution to Δm_K, $\Delta m_K < 10^{-16}$ MeV, which is below the uncertainty of the experimental measurement, $\Delta m_K = (3.490 \pm 0.006)\times10^{-12}$ MeV [22].

There is a formula for the the Δm_B mass difference similar to Eq. 55 from which one can obtain constraints on $(\delta_{13}^d)_{LR}$. $(\delta_{13}^d)_{LR}$ for the texture under consideration, Eq. 54, is given by,

$$(\delta_{13}^d)_{LR} = 3.5\times10^{-4}\left(\frac{A_b}{m_{\tilde{q}}}\right)\left(\frac{1\ \mathrm{TeV}}{m_{\tilde{q}}}\right)\frac{1}{t_\beta}. \quad (58)$$

Assuming $\tan\beta > 5$, $A_b < 4 m_{\tilde{q}}$, $m_{\tilde{q}} > 300$ GeV, and any gluino-squark mass ratio one obtains a contribution to Δm_B, $\Delta m_B < 7\times10^{-13}$ MeV, which is below the uncertainty of the experimental measurement, $\Delta m_B = (3.22 \pm 0.05)\times10^{-10}$ MeV [22]. Finally, from the measured $b \to s\gamma$ decay rate one can obtain limits on $(\delta_{23}^d)_{LR}$ through the formula [23],

$$B(b\to s\gamma) = \frac{2\alpha_s^2\alpha}{81\pi^2 m_{\tilde{q}}^4} m_b^3 \tau_b m_{\tilde{g}}^2 (\delta_{23}^d)_{LR}^2 M^2(x), \quad (59)$$

where $\alpha^{-1} = 127.934$, $\tau_b = 1.49\times10^{-12}$ s and the function $M(x)$ is defined in the appendix. For the texture under consideration, $(\delta_{23}^d)_{LR}$ is given by,

$$(\delta_{23}^d)_{LR} = 3.7\times10^{-2}\left(\frac{A_b}{m_{\tilde{q}}}\right)\left(\frac{1\ \mathrm{TeV}}{m_{\tilde{q}}}\right)\frac{1}{t_\beta}. \quad (60)$$

Assuming a large value of $\tan\beta$, $\tan\beta > 40$, $A_b \simeq m_{\tilde{q}}$, a gluino lighter than the squark $m_{\tilde{g}} < m_{\tilde{q}}$ and $m_{\tilde{q}} > 400$ GeV one obtains a contribution to $B(b\to s\gamma)$, $B(b\to s\gamma) < 3.4\times10^{-5}$, which is again below the uncertainty of the experimental measurement, $B(b\to s\gamma) = (3.3\pm0.4)\times10^{-4}$ [22] whitout requiring a very heavy squark spectra.

A. Contributions from flavor violating soft masses

We have shown that the radiative generation of quark masses and CKM elements does not necessarily implies overcoming constraints on flavor changing neutral current processes. On the other hand, we expect any theory of flavor generate certain amount of flavor violation or non-degeneracy in the soft mass matrices. Even whether the soft mass matrices are diagonal in the interaction basis they may not be diagonal in the SKM basis if the diagonal soft masses are not degenerate in the interaction basis. Since the most constrained entry is the (12) from the measurement of Δm_K, we will estimate what is the amount of nondegeneracy that we can afford between the first and generation of squark masses. For instance, the left handed down-type squark mass matrix in the SKM basis is given by,

$$(\mathcal{M}_{D_L}^2)^{\mathrm{SKM}} = (\mathcal{V}_L^d)^\dagger \mathcal{M}_{D_L}^2 \mathcal{V}_L^d, \quad (61)$$

and analogously for the right handed soft mass matrix. Assuming that $\mathcal{M}_{D_L}^2$ is diagonal, i.e. there is no flavor violation in interaction basis, and assuming that the non-degeneracy between first and second generation is,

$$\frac{(m_{\tilde{d}_L}^2 - m_{\tilde{s}_L}^2)}{m_{\tilde{d}_L}^2} = \lambda^n. \quad (62)$$

We obtain in the SKM basis

$$(\delta_{LL}^d)_{12} = \lambda^{n+1}, \quad (63)$$

and analogously for $(\delta_{RR}^d)_{12}$. This would generate a contribution to Δm_K [21] given by,

$$\Delta m_K = \frac{2\alpha_s^2}{648 m_{\tilde{q}}^2} m_K f_K^2 \lambda^{2(n+1)} \left[120\, xf(x) + 168\, g(x)\right.$$
$$\left.\left(\frac{m_K}{m_s+m_d}\right)^2 (384\, xf(x) - 24\, g(x))\right], \quad (64)$$

where we have added the LL and RR contributions assuming they are of the same size. The parameters in this formula where defined above. We obtain that for $n = 3$ we can avoid the saturation of the experimental measurement for squark spectra $m_{\widetilde{q}} > 400$ GeV, while for $n = 2$ the squark spectra must be $m_{\widetilde{q}} > 2$ TeV. This constraint can be milder if the gluino-squark mass ratio is much larger or smaller than one.

Using known expressions [21, 23] we can also calculate the size of a possible flavor violating soft mass to $B(b \to s\gamma)$. This is in general suppressed when compared with the LR, *i.e.* soft trilinear contribution. For instance for the LL contribution we obtain,

$$\frac{1}{6}\left(\frac{m_b}{m_{\widetilde{g}}}\right)\frac{(\delta^d_{12})_{LL}}{(\delta^d_{12})_{LR}} \approx 3\times 10^{-3} t_\beta \left(\frac{m_{\widetilde{b}}}{m_{\widetilde{g}}}\right) \tag{65}$$

where we assumed that $(\delta^d_{12})_{LL} = \lambda$. We used that v and γ_τ are given by $v = 175$ GeV and $\gamma_b \approx 0.1$. Even considering very large $\tan\beta$ values a possible LL contribution is one order of magnitude smaller than the LR contribution. Therefore the approximate constraints on the supersymmetric spectra calculated above from the LR contribution to $B(b \to s\gamma)$, while ignoring the LL contributions, are still valid.

To sum up, the flavor violation present in the soft trilinear supersymmetry breaking sector, which is necessary in this scenario to generate quark mixings radiatively, is not excluded by the present experimental constraints on FCNCs processes. The approximate radiative alignment between Yukawa matrices and soft trilinear terms helps to suppress some of the supersymmetric contributions to FCNCs. On the other hand if the underlying theory of flavor predicts non–degeneracy between the masses of first and second generation of squarks larger than λ^2 the constraints on the squark spectra, coming from the measurement of Δm_K, are much stronger than the ones from the LR contributions.

IV. CHARGED LEPTON MASSES

The electron and muon masses could also be generated radiatively through one–loop bino–slepton exchange involving the soft supersymmetry-breaking terms, analogously to the gluino–squark exchange in the quark sector. This possibility that was first suggested in Refs. [5, 9]. The radiatively generated lepton Yukawa couplings are in this case given by,

$$(\mathbf{Y}_L)^{\rm rad}_{ab} = \frac{\alpha}{2\pi} m_{\widetilde{\gamma}} \sum_c \mathcal{Z}^L_{ac}\mathcal{Z}^{L*}_{(b+3)c} B_0(m_{\widetilde{\gamma}}, m_{\widetilde{l}_c}), \tag{66}$$

where $m_{\widetilde{\gamma}}$ is the photino mass, $\mathcal{Z}^L$ is the slepton diagonalization matrix and $m_{\widetilde{l}_c}$ $(c = 1, \cdots 6)$ are slepton mass eigenstates.

To make my case I will pick a simple texture for the soft trilinear matrix $\mathbf{A}_L$, which is motivated on a SU(5) unified model plus a $\mathrm{U}(2)_H$ horizontal symmetry [15, 16],

$$\mathbf{A}_L = A_\tau \left[\begin{array}{ccc} 0 & \lambda_l^2 & \lambda_l^2 \\ \lambda_l^2 & 3\lambda_l & \lambda_l \\ \lambda_l^2 & \lambda_l & 1 \end{array}\right]. \tag{67}$$

I assume that first and second generation slepton masses are degenerate and I allow non–degeneracy between third and first/second generation, as in the squark sector I will parametrize the non–degeneracy through the coefficient κ analogous to the one defined by Eq. 14. One obtains then a simple expression for the radiatively corrected lepton quark mass matrix,

$$\mathbf{m}_L = \widehat{m}_\tau \left[\begin{array}{ccc} 0 & \gamma_\tau\lambda_l^2 & \kappa\gamma_\tau\lambda_l^2 \\ \gamma_\tau\lambda_l^2 & 3\gamma_\tau\lambda_l & \kappa\gamma_\tau\lambda_l \\ \kappa\gamma_\tau\lambda_l^2 & \kappa\gamma_\tau\lambda_l & 1 \end{array}\right], \tag{68}$$

In the $m_{\widetilde{l}} \geq m_{\widetilde{B}}$ limit one obtains $\kappa = \sigma^2\ln\sigma^2/(\sigma^2-1)$. $\widehat{m_\tau}$ is defined by,

$$\widehat{m}_\tau = v c_\beta \left(y_\tau + \rho_L(1 - \frac{\mu}{A_\tau} y_\tau \tan\beta)\right), \tag{69}$$

γ_τ is defined by an expression similar to the one for $\gamma_{b,t}$,

$$\gamma_\tau = \frac{v c_\beta \rho_L}{\widehat{m}_\tau}, \tag{70}$$

In the case $m_{\widetilde{l}} \geq m_{\widetilde{B}}$ and $\sigma \lesssim 2$ one obtains,

$$\rho_L = \frac{\alpha}{\pi}\left(\frac{m_{\widetilde{\gamma}}}{m_{\widetilde{\tau}}}\right)\left(\frac{A_\tau}{m_{\widetilde{\tau}}}\right)\left(\frac{1}{\sigma^2}\right), \tag{71}$$

(for $\sigma > 2$ one should substitute $\sigma \to 2\ln\sigma$). Although not diagonal in the gauge basis the matrix $\mathbf{m}_L$ can be brought to diagonal form in the mass basis by a biunitary diagonalization, $(\mathcal{V}^l_L)^\dagger \mathbf{m}_L \mathcal{V}^l_R = (m_e, m_\mu, m_\tau)$. The lepton mass matrix given by Eq. 68 makes the following predictions for the quark mass ratios,

$$\frac{m_e}{m_\mu} = \frac{1}{9}\lambda_l^2(1 + \frac{5}{3}\gamma_\tau\lambda_l - \frac{2}{9}\lambda_l^2) + \mathcal{O}(\lambda_l^4), \tag{72}$$

$$\frac{m_\mu}{m_\tau} = 3\gamma_\tau\lambda_l(1 - \frac{1}{3}\gamma_\tau\lambda_l + \frac{1}{9}\lambda_l^2) + \mathcal{O}(\lambda_l^3), \tag{73}$$

which approximately reduce to,

$$\frac{m_e}{m_\mu} = \frac{1}{9}\lambda_l^2, \quad \frac{m_\mu}{m_\tau} = 3\gamma_\tau\lambda_l. \tag{74}$$

We can express λ_l and γ_τ as a function of dimensionless and approximately renormalization scale independent charged lepton mass ratios, to first order,

$$\lambda_l = 3\left(\frac{m_e}{m_\mu}\right)^{1/2}, \quad \gamma_\tau = \frac{1}{9}\left(\frac{m_\mu^3}{m_\tau^2 m_e}\right)^{1/2}, \tag{75}$$

using the invariant running lepton mass ratios determined from experiment (see appendix). In the degenerate limit, *i.e.* $\kappa = 1$, one obtains,

$$\lambda_l = 0.206480 \pm 0.000002, \tag{76}$$

$$\gamma_\tau = 0.09495 \pm 0.00001, \tag{77}$$

Interestingly these values of λ_l and γ_τ are consistent with the values required in the quark sector for $\lambda_{u,d}$ and $\gamma_{t,b}$ respectively, unveiling two surprising relations,

$$\lambda = \left(\frac{m_d}{m_s}\right)^{1/2} \approx \left(\frac{m_u}{m_c}\right)^{1/4} \approx 3\left(\frac{m_e}{m_\mu}\right)^{1/2}, \tag{78}$$

$$\theta = \left(\frac{m_s^3}{m_b^2 m_d}\right)^{1/2} \approx \left(\frac{m_c^3}{m_t^2 m_u}\right)^{1/2} \approx \frac{1}{9}\left(\frac{m_\mu^3}{m_\tau^2 m_e}\right)^{1/2} \tag{79}$$

where λ is related to the Cabbibo angle and θ is a new parameter. These coincidences indicate that λ and θ are parameters directly connected with the underlying theory of flavor. The coincidence of these mass ratios may be considered experimental evidence supporting the consistency of this scenario. Using Eq. 70 and assuming that the slepton masses are heavier than the bino mass, $m_{\widetilde{e}} \geq m_{\widetilde{B}}$, one obtains the following constraint for the non-degeneracy between first and third generations slepton masses,

$$\sigma = \frac{1}{\gamma_\tau}\left(\frac{vc_\beta}{m_\tau}\right)\left(\frac{\alpha}{\pi}\right)\left(\frac{m_{\widetilde{B}}}{m_{\widetilde{e}}}\right)\left(\frac{A_\tau}{m_{\widetilde{\tau}}}\right) \lesssim \frac{5}{t_\beta}. \tag{80}$$

This would imply a considerable amount of non-degeneracy in the slepton sector between the first/second and third generations, $m_{\widetilde{\tau}} \geq \frac{1}{5}\, t_\beta\, m_{\widetilde{e}}$. On the other hand if the bino mass is heavier than the selectron mass the required slepton non–degeneracy is reduced. It would be interesting if an alternative soft trilinear lepton texture could be found that keeps the succesfull prediction given by, $\lambda_l = 3\left(\frac{m_e}{m_\mu}\right)^{1/2}$, without requiring slepton non-degeneracy. It would also be interesting if the corrections to the supersymmetry breaking sector could generate the important slepton non-degeneracy that seems to be required in the slepton sector by the texture here examined. Nevertheless, we would like to mention that there is an alternative possibility that could allow us to fit the lepton mass ratios without resorting to a highly non-degenerate slepton spectra.

A. Non-holomorphic terms

The most general softly broken supersymmetric lagrangian can contain non-holomorphic operators of the form [11],

$$\frac{1}{M^3}\mathcal{Z}\mathcal{Z}^\dagger H_\alpha^\dagger \phi_L \phi_R, \tag{81}$$

where $\mathcal{Z}$ are the supersymmetry- and flavor-breaking chiral superfields. These terms are suppressed for a messenger scale well above the supersymmetry breaking scale. They can therefore be relevant only with a low scale for both flavor and supersymmetry breaking. When they are relevant, they would give rise to additional soft trilinear terms relevant for the radiative Yukawa generation in the down-type quark and lepton sectors. For instance, in the slepton sector of the soft breaking lagrangian there is an additional contribution,

$$A_\tau \mathcal{H}_d L E + A'_\tau \mathcal{H}_u^* L E + \cdots \tag{82}$$

The calculation of the radiative masses is then similar to the one implemented above except for a shift in the soft trilinear terms by an amount,

$$A_\tau \to A_\tau + \tan\beta A'_\tau, \tag{83}$$

where the $\tan\beta$ factor comes from the H_u^* term. The non-holomorphic contribution, A'_τ, is dominant for large $\tan\beta$. In such a case non-degeneracy in the slepton sector would not be required to fit the lepton masses.

B. Lepton flavor violating processes

Overcoming the present experimental constraints on supersymmetric contributions to lepton flavor changing processes is a necessary requirement for the consistency of our scenario [19]. As in the squark sector, for calculational purposes, it is convenient to rotate the sleptons to the basis where gaugino vertices are flavor diagonal [21]. The soft trilinear matrix $\mathbf{A_L}$ in the superKM basis is given by,

$$\mathbf{A}_L^{\rm SKM} = (\mathcal{V}_L^l)^\dagger \mathbf{A}_L \mathcal{V}_R^l. \tag{84}$$

Assuming the soft trilinear texture from Eq. 67, one obtains, to leading order in λ_l and γ_τ,

$$\left|\mathbf{A}_L^{\rm SKM}\right| = A_\tau \begin{bmatrix} \frac{1}{3}\lambda_l^3 & \frac{2}{3}\gamma_\tau\lambda_l^3 & \frac{2}{3}\lambda_l^2 \\ \frac{2}{3}\gamma_\tau\lambda_l^3 & 3\lambda_l & \lambda_l \\ \frac{2}{3}\lambda_l^2 & \lambda_l & (1+2\gamma_\tau\lambda_l^2) \end{bmatrix}, \tag{85}$$

Using for λ_l and γ_τ the values given by Eqs. 76 & 77 the amount of soft flavor violation is determined,

$$\left|\mathbf{A}_L^{\rm SKM}\right| = A_\tau \begin{bmatrix} 310^{-3} & 5.6\times10^{-4} & 3\times10^{-2} \\ -- & 0.648 & 0.216 \\ -- & -- & 1.007 \end{bmatrix}, \tag{86}$$

The entry most constrained experimentally in the soft trilinear mass matrix is the entry (12). Its contribution to the $B(\mu\to e\gamma)$ is, following Ref. [21], given by,

$$\Gamma_{\mu\to e\gamma} = \frac{B(\mu\to e\gamma)}{B(\mu\to e\nu_\mu\overline{\nu_e})} = \frac{24\alpha^3\pi}{m_\mu^2 G_F^2 m_{\widetilde{l}}^4} m_{\widetilde{\gamma}}^2 (\delta_{12}^l)_{LR}^2 M^2(x) \tag{87}$$

where $\alpha^{-1} = 127.934$, $G_F = 1.16639\times10^{-11}$ MeV^{-2} and the function $M(x)$ is defined in the appendix. For the texture under consideration $(\delta_{12}^l)_{LR}$ is given by,

$$(\delta_{12}^l)_{LR} = 10^{-4}\left(\frac{A_\tau}{m_{\widetilde{l}}}\right)\left(\frac{1\ \text{TeV}}{m_{\widetilde{l}}}\right)\frac{1}{t_\beta}. \tag{88}$$

Assuming a large value of $\tan\beta > 50$, $A_b \simeq m_{\widetilde{l}}$, a photino mass $m_{\widetilde{\gamma}} < m_{\widetilde{l}}$ and $m_{\widetilde{l}} > 1$ TeV one obtains a branching fraction $\Gamma_{\mu\to e\gamma} < 8\times 10^{-12}$, which is still below the experimental limit, $\Gamma^{\rm exp}_{\mu\to e\gamma} < 1.2\times 10^{-11}$ [22], without requiring a multiTeV slepton spectra. On the other hand, this indicates that unless the supersymmetric spectra is above 1 TeV this process could be observed in the near future. A possible contribution to $B(\mu\to e\gamma)$ from flavor violating soft masses of the order $(\delta^l_{12})_{LL} \approx \lambda^3$ would receive a suppression factor compared with the contribution from the soft trilinear terms of the form,

$$\frac{1}{6}\left(\frac{m_\mu}{m_{\widetilde{\gamma}}}\right)\frac{(\delta^l_{12})_{LL}}{(\delta^l_{12})_{LR}} \approx 5\times 10^{-3} t_\beta \left(\frac{m_{\widetilde{l}}}{m_{\widetilde{\gamma}}}\right). \qquad (89)$$

where we used that v and γ_τ are given by $v = 175$ GeV and $\gamma_\tau = 0.95$. The predictions for $\Gamma_{\tau\to e\gamma}$ and $\Gamma_{\tau\to\mu\gamma}$ can be calculated from Eq. 87 with the substitutions $m_\mu \to m_\tau$ and $\delta^l_{12} \to \delta^l_{13}, \delta^l_{23}$ respectively. For the texture under consideration, using the same parameter space limits indicated above, I obtain, $\Gamma_{\tau\to e\gamma} < 8\times 10^{-11}$ and $\Gamma_{\tau\to\mu\gamma} < 4\times 10^{-10}$, these two are far below the present experimental limits, $\Gamma^{\rm exp}_{\tau\to e\gamma} < 2.7\times 10^{-6}$ and $\Gamma^{\rm exp}_{\tau\to\mu\gamma} < 1.1\times 10^{-6}$ [22].

C. Massless neutrinos

The generation of small neutrino masses required by the experiment is an open problem for this scenario. It is not possible to generate radiatively a Majorana or Dirac neutrino Yukawa matrix through this mechanism in the MSSM, since even if a right-handed neutrino is introduced it cannot give rise to a radiatively generated Dirac Yukawa matrix, because it carries no $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ quantum numbers. The solution to this problem most probably will require to enlarge the particle content and the symmetries of the model.

V. PROTON DECAY SUPRESSION

The mechanism of soft radiative generation of Yukawa couplings could be embedded in the particular case of a supersymmetric grand unified theory. It is believed that strong experimental limits on proton decay place stringent constraints on supersymmetric grand unified models. This assertion, however, is very dependent on the Yukawa structure of the supersymmetric theory. For instance, in the case of minimal supersymmetric SU(5) model [28] the superpotential, omitting SU(5) and flavor indices, is given by

$$\mathcal{W}_{\rm SU(5)} = \frac{1}{4}\lambda_U \widehat{\psi}_{10}\widehat{\psi}_{10}\widehat{\mathcal{H}}_5 + \sqrt{2}\lambda_D \widehat{\psi}_{10}\widehat{\psi}_{\overline{5}}\widehat{\mathcal{H}}_{\overline{5}} + \cdots \qquad (90)$$

where $\widehat{\psi}_{10}$ and $\widehat{\psi}_{\overline{5}}$ are matter chiral superfields belonging to representations **10** and $\overline{\mathbf{5}}$ of SU(5), respectively. As in the supersymmetric generalization of the SM, to generate fermion masses we need two sets of Higgs superfields, $\widehat{\mathcal{H}}_5$ and $\widehat{\mathcal{H}}_{\overline{5}}$, belonging to representations **5** and $\overline{\mathbf{5}}$ of SU(5). In ordinary SUSY GUTs, after integrating out the colored Higgs triplet, the presence of Yukawa couplings in the superpotential leads directly to effective dimension-five interactions which omitting flavor indices are of the form,

$$\begin{aligned}\mathcal{W}_{\dim 5} \;=\; & \frac{1}{M_{\mathcal{H}_c}}\left[\frac{1}{2}\lambda_U\lambda_D\,(QQ)\,(QL) + \right.\\ & \left.\lambda_U\lambda_D\,(UE)\,(UD)\right], \qquad (91)\end{aligned}$$

where the operators $(QQ)\,(QL)$ and $(UE)\,(UD)$ are totally antisymmetric in color indices. Therefore flavor conservation in the superpotential would imply their cancellation,

$$(QQ)\,(QL) \;\equiv\; 0, \qquad (92)$$
$$(UE)\,(UD) \;\equiv\; 0. \qquad (93)$$

In our case, we started assuming that there is a symmetry that guarantees flavor conservation in the superpotential of the supersymmetric unified theory, as is expected from any model generating flavor radiatively. Flavor violating Yukawa couplings are only generated at low energy after supersymmetry breaking. We can see that after integrating out coloured Higgsses one can generate operators generically of the form,

$$\frac{1}{M^2}(\mathcal{Z}\mathcal{Z})QQQL.$$

These operators cannot generate directly dimension five operators because we assumed that only the auxiliary components of the SUSY breaking $\mathcal{Z}$-fields acquire a vev, breaking the flavor symmetry. Dimension five operators could be generated at higher orders. Since tree level interactions with coloured Higgsinos are only possible for the third family, the generation of a dimension five proton decay operator would require two flavor mixing couplings between first and third generation. On the other hand, the Yukawa coupling of the form $(\mathbf{Y}_U)_{13}$ is first generated at two loops and is very suppressed, as pointed out in Eq. 51. As a consequence radiatively generated dimension five operators leading to proton decay are extremely suppressed in this scenario, when compared with ordinary SUSY GUT predictions, which generate flavor in the superpotential. Regarding the next dominant decay mode coming from dimension-six operators via GUT gauge bosons. It has been shown that using the SuperKamiokande limit, $\tau(p\to\pi^0 e^+) > 5.3\times 10^{33}$ years, a lower bound on the heavy gauge boson mass, M_V, can be extracted, $M_V > 6.8\times 10^{15}$ GeV. Furthermore, the proton decay rate for $M_V = M_{GUT}$ is far below the detection limit that can be reached within the next years [29].

VI. CP VIOLATION

The CP violation experimentally observed in the SM has not been included in this analysis. There is no doubt that CP violation can be accommodated in this scenario since the supersymmetry breaking sector provides numerous sources of CP violation. If that is the case, we expect correlations between supersymmetric contributions to processes such as: $K^0 - \overline{K}^0$ mixing, electric dipole moments, $B \to \phi K_S$, etc. The extension of this scenario to include CP-violation will be the subject of a future paper.

Appendix

For the calculation of the fermion mass ratios in the main text running quark masses were used. These were calculated through scaling factors including QCD and QED renormalization effects, which can be determined using known solutions to the SM RGEs. For the charged leptons our starting point are the well known physical masses. For the top quark the starting point is the pole mass from the PDG collaboration [22],

$$m_t \;=\; 174.3 \pm 5.1 \text{ GeV}. \tag{94}$$

For the bottom and charm quarks the running masses, $m_b(m_b)_{\overline{MS}}$ and $m_c(m_c)_{\overline{MS}}$ from Refs. [24] & [25] are used,

$$\begin{aligned} m_b(m_b)_{\overline{MS}} &= 4.25 \pm 0.25 \text{ GeV}, & (95)\\ m_c(m_c)_{\overline{MS}} &= 1.26 \pm 0.05 \text{ GeV}. & (96)\end{aligned}$$

For the light quarks, u,d and s, the starting point is the normalized $\overline{MS}$ values at $\mu = 2$ GeV. Original extractions [26, 27] quoted in the literature have been rescaled as in [22],

$$\begin{aligned} m_s(2\text{ GeV})_{\overline{MS}} &= 117 \pm 17 \text{ MeV}, & (97)\\ m_d(2\text{ GeV})_{\overline{MS}} &= 5.2 \pm 0.9 \text{ MeV}, & (98)\\ m_u(2\text{ GeV})_{\overline{MS}} &= 2.9 \pm 0.6 \text{ MeV}. & (99)\end{aligned}$$

For completeness we also include here some functions used in the main text. The B_0 function used in the calculation of the one–loop finite corrections is given by

$$B_0(m_1, m_2) = 1 + \ln\left(\frac{Q^2}{m_2^2}\right) + \frac{m_1^2}{m_2^2 - m_1^2}\ln\left(\frac{m_2^2}{m_1^2}\right). \tag{100}$$

The following functions, extracted from Ref. [21], are used in the calculation of Δm_K, Δm_B and $B(b \to s\gamma)$,

$$\begin{aligned} f(x) &= \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5}, & (101)\\ g(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}, & (102)\\ M(x) &= \frac{1 + 4x - 5x^2 + 4x\ln x + 2x^2\ln x}{2(1-x)^4}. & (103)\end{aligned}$$

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